Team Control Number

12678

Problem Chosen

B

2022
HiMCM
Summary Sheet

Climate change is one of the most pressing issues in our society today. Due to an increased need for energy, humans have been burning copious amounts of fossil fuels such as oil and coal, unnaturally releasing carbon that has been trapped in the earth for millions of years. As a result, the carbon cycle—Earth's natural process of keeping carbon balanced throughout the ocean, atmosphere, and biosphere—is disrupted due to this abundance of carbon dioxide (CO_2) trapping excess heat in our atmosphere. Consequently, CO_2 in our atmosphere has reached its highest point in the last 3.6 million years, and correspondingly, global temperatures are increasing year by year [1]. Thankfully, only a fraction of carbon emitted accumulates in the atmosphere. Earth has many carbon sinks: reservoirs that absorb and store carbon dioxide. It is estimated that 54 percent of total CO_2 emissions are absorbed by the land and ocean carbon [2]. While these carbon sinks absorb a large percentage of emissions, human emissions far surpass the amount which can be absorbed by the carbon sinks, disrupting the natural carbon cycle. With so many variables at play beyond just carbon sinks, predicting atmospheric CO_2 levels becomes a daunting task. In this paper, we outline 3 mathematical models of CO_2 of varying complexities, in order to project future CO_2 values and respond to OECD's prediction of a CO_2 level of 685 ppm by 2050.

To start, we created a simple linear regression model, derived from the data given on the problem sheet. This model is effective at giving a general prediction that is more or less around the desired range. However it fails to take into account any actual factors that may affect CO_2 emissions. In order to reflect more accurately actual real world trends, we created a model considering GDP growth and carbon intensity. Since the main driving force behind carbon emission is industrialization and economic growth, adding GDP increases the model's real life application. Lastly, we built a mechanistic model which considered the diminishing effectiveness of land and carbon sinks, while retaining the features of the previous models.

We then utilized these models to study the scenario where carbon control policies are implemented globally. Our model results show that if human beings take actions to control carbon emissions, CO_2 concentration and temperature would eventually decrease. The climate change crisis could be controlled or mitigated. Without any actions, CO_2 concentration and temperature would continuously increase.

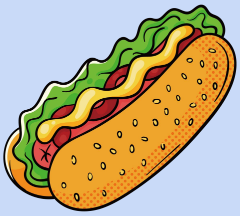
Key Words: Carbon Cycle, Carbon Sink, Global Warming, Predicted Carbon Level

Carbon Emissions Models: Marvelous or Perilous?

#12678

Three new models for predicting carbon dioxide concentrations offer a new look on our world's future

With carbon emissions and global warming being ever-pressing problems for the world, it is crucial that we are able to predict future CO2 concentration and temperature levels in the atmosphere. Given a set of historical CO2 concentration data and research on other possible influential factors, we were able to construct 3 mathematical models of varying complexities.We sum up the creation of our models using the analogy of a perfectly grilled hotdog.

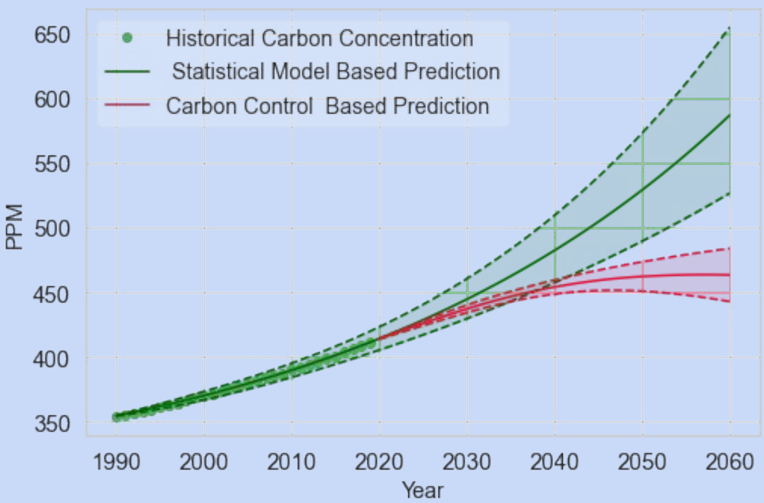


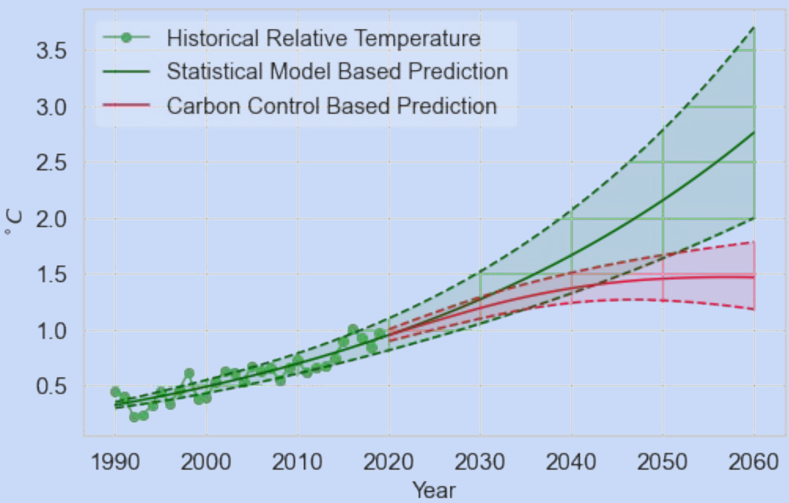
The first model acts like the bun: the bare skeleton for each model that we build on. We constructed a simple model using only past data fitted into a linear line. Although the predictions were reasonable, we suspected that there may have be many underlying factors that would affect CO2 concentration. The bun was missing a crucial element.

To add on to the bare bun of the hotdog, we created a statistical model based on economic factors which could drastically affect CO2 emissions and, in turn, CO2 concentration. Keeping in mind the main contributor of CO2 emissions is human usage of fossil fuels, the root cause can be traced back to a need for energy. Since energy usage increases with human economic and industrial activity, a measure of human economic and industrial activity should correlate with carbon emissions. Therefore, we chose to use GDP and carbon intensity (CO2 emissions per dollar GDP adjusted for inflation) in order to create our second model, which could be thought of as the sausage and the bun: a delicious combination, but still has room for improvement.

Finally, we add the finishing embellishments to the hotdog. We get a more complete taste profile by adding condiments and toppings. The mechanistic model incorporates the previous statistical model, but takes into consideration the different rates at which the planet can naturally absorb CO2. Our model reflects how the environment's ability to absorb CO2 will change in the future, due to our complex natural ecosystems responding to the changing CO2 concentrations.

We applied our models to a simple scenario where major countries are able to successfully implement their carbon control policies. In addition, our CO2 predictions can be used to predict temperature as there exist a clear correlation.





Green line: signifies CO2 and temperature trends without implementation
Red line: signifies successful implementation of carbon control policies

Conclusion:

Our analysis indicates that with carbon control policies, CO2 concentration and temperature would eventually decrease, successfully combating climate change. In order to do so, we recommend an emphasis on climate mitigation through reducing reliance on fossil fuels and increasing energy efficiency. Additionally, increasing reforestation and decreasing deforestation will help to increase the environment's natural ability to absorb CO2. However, without any initiative to control CO2, climate change would be inevitable.

Contents

1 Introduction

1.1 Background

Global carbon emission level began its dramatic growth since the start of the industrial revolution in the 1800s, starting with massive expansion of cities and human populations around the world. However, global carbon emissions had not accelerated to the rate that it is growing today until the 1950s, when the massive energy potential of burning fossil fuels was realized and exploited by manufacturers. Since then, the burning of fossil fuels has been the backbone of urbanization and population growth. However, this growth also means that the atmosphere is accumulating unprecedented amounts of carbon dioxide. This is harmful as carbon dioxide traps heat in our atmosphere as the global warming effect, causing climate change that includes warming temperature, rising sea level, increasing extreme weather events, etc.

The global carbon cycle is a complicated system (see Figure 1). After CO_2 is released to the atmosphere due to the burning of fossil fuels, some stay in the atmosphere while the others are absorbed by land and ocean sinks. The land and ocean sinks could change associated with the area and capacity of biological absorption in land and ocean. This delicate balance makes it difficult to understand the dynamic change of CO_2 concentration in the atmosphere, even though we can relatively easily estimate the global consumption of fossil fuels. Therefore, predicting atmospheric CO_2 concentration and correspondingly global temperature is challenging.

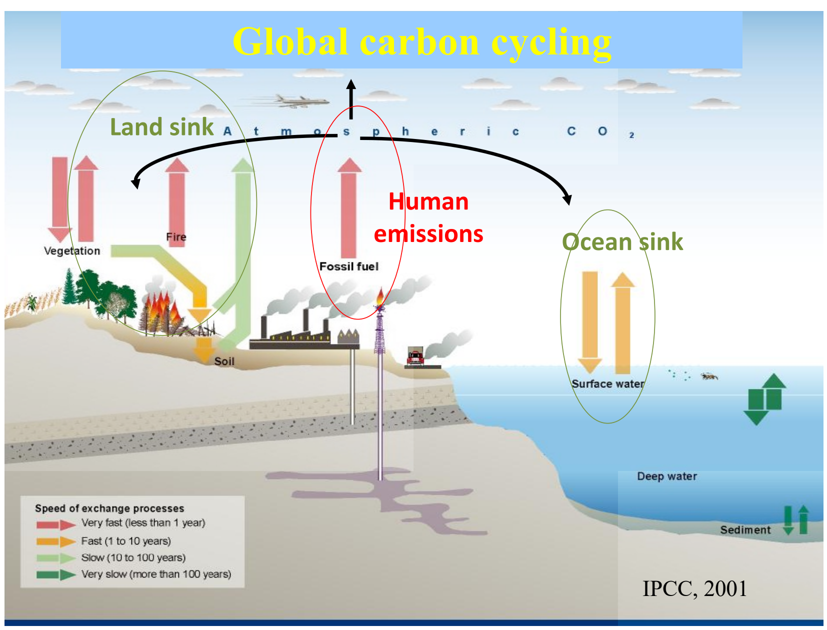


Figure 1: Global carbon cycling

Increasing global temperature cascades into a myriad of effects on both nature and humans: notably, rising global sea levels, among other consequences. Glaciers make up 2.1 percent of earth's water supply. Rising temperatures causes these glaciers to melt, leading to a rising sea level rate of 3.4 millimeters per year [3]. Hundreds of millions people are estimated to be forced to relocate in the coming decades [4]. With the aforementioned consequences of excess CO_2 and its effect of global warming, continuing our current emission quantity would be detrimental to humanity and the earth. Therefore, it is crucial that we have accurate predictions of what carbon dioxide levels will be like in the future so that we can take immediate action to alleviate or even eliminate this problem. Our following mathematical models look to make an accurate estimation that considers various nuanced factors to create effective predictions.

1.2 Section Summaries

The rest of this paper is organized as follows. Section 2 discusses the relationship between atmospheric CO_2 concentration and temperature and how we can use concentration to predict the temperature. Sections 3 through 5 explain our models and aim to answer this paper's questions about predicting future temperature and carbon emissions. Section 6 offers a different scenario in which the global top 3 emitters follow through with their carbon emission reduction plans. Finally, in section 7, we summarize our findings about each model and how each model is best applied.

2 CO_2 Concentration and Temperature

There is a wide consensus in the scientific community that carbon emissions cause the increase of the atmosphere CO_2 concentration, which then causes global warming. Greenhouses gases like carbon dioxide are an essential component of our atmosphere because they absorb energy from the sun; without them, the earth would not be warm enough to sustain life. However, too much heat is trapped by carbon dioxide when there is an excess amount of it in our atmosphere, thereby increasing the temperature of the earth.

In this section, we first investigate the relationship between the atmosphere CO_2 concentration and temperature and and build a model to predict the temperature from the CO_2 concentration. Notice that both CO_2 concentration and temperature depend on the measurement time and location, we have to define them more carefully. In this report, we use the annual month of March averages of CO_2 expressed as a mole fraction in dry air (parts per million, ppm) derived from continuous air samples for the Mauna Loa Observatory, Hawaii, U.S.A. as the measurement of the *annual CO_2 concentration*, denoted by C_t; and use the global annual mean surface-air temperature change in degrees Celsius based on land and ocean data compared to the temperature mean of the base period 1951-1980 as the *annual relative temperature*, denoted by T_t.

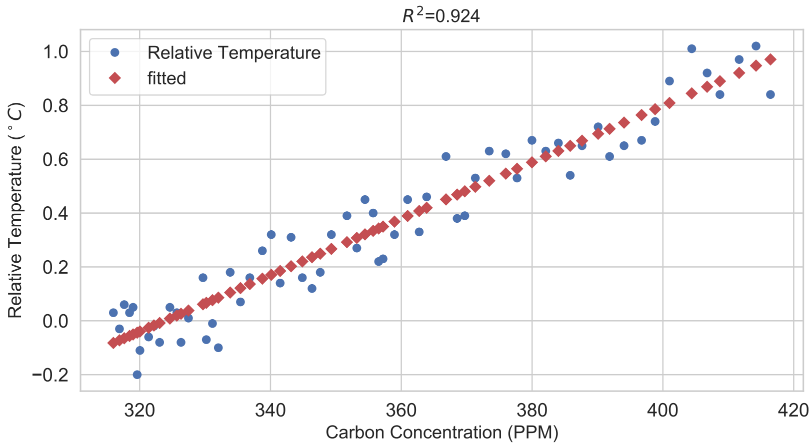


Figure 2: Linear model of concentration to temperature

We plot the annual CO_2 concentrations versus the annual relative temperature from 1959 to 2021 in Figure 2. It is clear that a linear relationship might fit the data well. Therefore, we conduct a linear regression and the resulted equation is as follows:

$$T_t = -3.3926 + 0.0105\, C_t, \tag{1}$$

which is also shown on Figure 2. The detailed report from the linear regression analysis is shown in Figure 3. From the report, we see that the R^2 value is 0.924 and the p-values (a measurement used to find statistical significance, where a lower value is desirable) of the coefficients are $2.206\,206 \times 10^{-33}$ and $7.340\,353 \times 10^{-36}$, indicating that the model fits the data appropriately. This establishes that we can appropriately predict the annual relative temperature with the annual CO_2 concentration data, setting a strong base for our three models that come later.

3 Model 1 - A Simple Model

3.1 What is the simple model?

We first consider a simple time-series model of the annual CO_2 concentration without considering any potential impacting variables. We plot the annual CO_2 concentration versus year in the left panel of Figure 4. There is a clear nonlinear behavior in the data. Furthermore, as the annual changes of the concentration are quite small relative to the values of concentrations, directly fitting the concentrations may result in misleading results. Therefore, we consider the annual changes of the CO_2 concentration, defined as $\Delta C_t = C_t - C_{t-1}$, and plot them in the right panel of Figure 4.

```
                 OLS Regression Results
==============================================================================
Dep. Variable:    Relative Temperature  R-squared:                 0.924
Model:                       OLS  Adj. R-squared:            0.923
Method:             Least Squares  F-statistic:                 743.0
Date:             Sun, 13 Nov 2022  Prob (F-statistic):         7.34e-36
Time:                     16:45:24  Log-Likelihood:             63.093
No. Observations:               63  AIC:                        -122.2
Df Residuals:                   61  BIC:                        -117.9
Df Model:                        1
Covariance Type:         nonrobust
==================================================================================
                  coef    std err        t     P>|t|     [0.025     0.975]
----------------------------------------------------------------------------------
const              -3.3926    0.138   -24.618    0.000    -3.668    -3.117
Carbon Concentration   0.0105   0.000   27.259   0.000    0.010     0.011
==============================================================================
Omnibus:                 11.338  Durbin-Watson:              1.669
Prob(Omnibus):            0.003  Jarque-Bera (JB):           3.215
Skew:                    -0.034  Prob(JB):                   0.200
Kurtosis:                 1.895  Cond. No.                4.34e+03
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Figure 3: Regression report of concentration to temperature

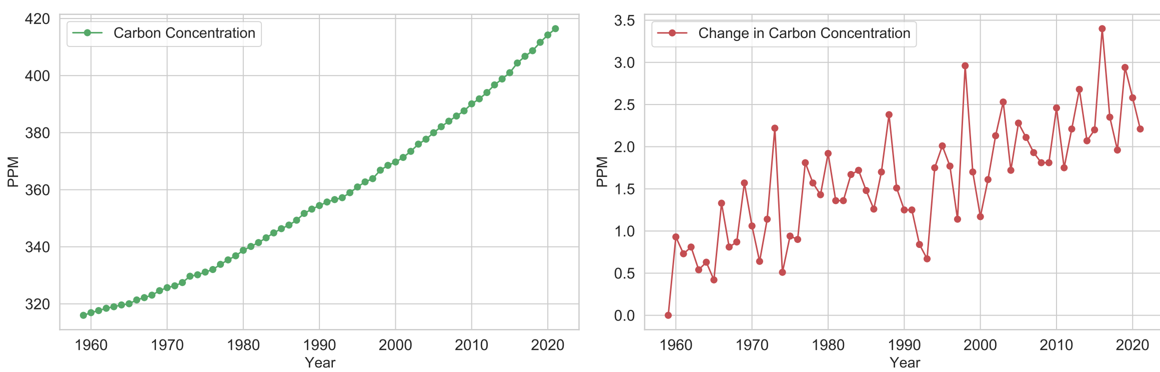

Figure 4: Carbon concentrations and their differences

Following the theory of Ockham's razor, we decide that a linear model is valuable as it effectively models the data with the fewest parameters. Therefore, we make the following assumption.

Assumption 1. *ΔC_t is linear in t, i.e., $\Delta C_t = \beta_0 + \beta_1 t$ for some β_0 and β_1.*

3.2 Results from the simple model

We conduct a linear regression analysis of ΔC_t and t, the fitted line is shown in Figure 5. The resulted equation from the linear regression is

$$\Delta C_t = -54.4404 + 0.0282\, t. \tag{2}$$

Even though the R^2 value is 0.569, which is not high, the p-values of the coefficients are very close to 0, suggesting that this linear model still has statistical significance despite the low R^2 value, which may be caused by the randomness in the data.

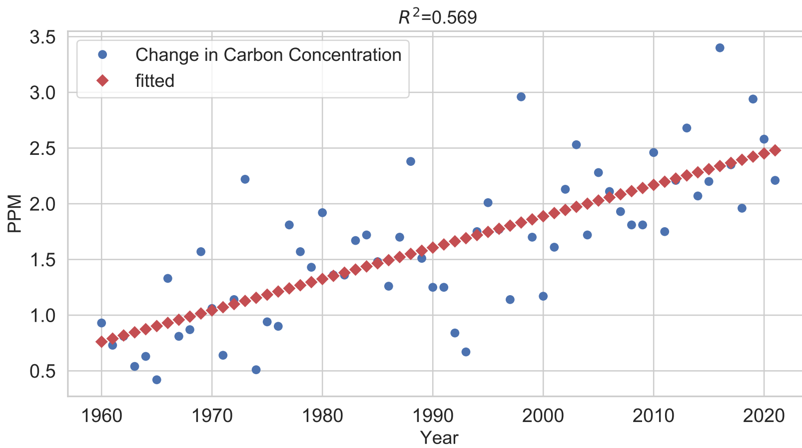

Figure 5: Linear model of ΔC_t to t

From Equation (2), we can calculate C_t recursively with the equation:

$$C_t = C_{1959} + \sum_{i=1959}^{t} \Delta C_t.$$

Together with Equation (2), we can predict the CO_2 concentrations in the future. Furthermore, because of the randomness in the data, we also calculate the upper bounds and lower bounds of the 95% confidence intervals of the predicted values. These result in three distinct curves in Figure 6. By utilizing the mean value and the upper and lower intervals of the predicted ΔC, we can find an range of values where C_t is more likely to be. This helps the model account for randomness, giving us a better picture of the possibilities of the future CO_2 concentrations.

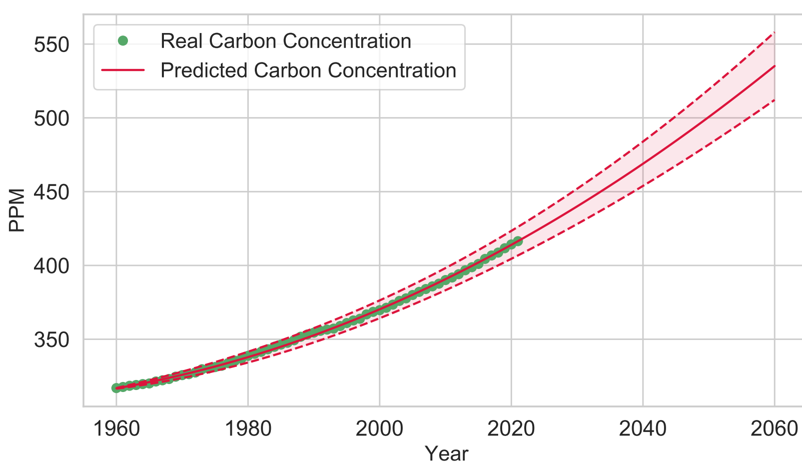

Figure 6: Predicted CO_2 concentration

The simple model predicts that the CO_2 concentration of 2050 is in the range of 481.93 to 519.28 ppm with a mean value of 500.61 ppm, and that of 2010 is in the range of 655.16 to 747.50 ppm with a mean value of 701.32 ppm.

Clearly, this model differs massively from the OECD's prediction of 685 ppm by 2050. In fact, the simple model projects the CO_2 concentration to hit 685 ppm between 2089 and 2108, decades after 2050. We believe that the OECD's model may be much more sophisticated than this simple model, considering many additional factors that could affect CO_2 concentration. However, our predicted concentrations are still very reasonable, as many other reputable studies predicted the 2050 concentrations in the 500's ranges [5].

In one of the question that this paper aims to answer, we are asked *whether the March 2004 increase of CO_2 resulted in a larger increase than observed over any previous 10-year period*. Even though the observed March 2004 CO_2 increase is only 1.72 that is below the increases in 2002 and 2003, We believe it is a larger increase. This is because the overall data in this model suggests that ΔC_t follows an increasing trend and therefor we can attribute the observed low value of ΔC_{2004} to the randomness in the data.

3.3 Temperature predictions from the simple model

With the predicted CO_2 concentrations from the simple model, we can use the model between the CO_2 concentration and relative temperature (Equation 1) established in Section 2 to predict the relative temperatures in the future. We take the mean values and the upper and lower bounds of the predicted concentrations to calculate the mean values and the upper and lower bounds of the predicted relative temperatures, respectively. This is illustrated in Figure 7.

3.3.1 Predicted years to reach $1.5°C$ and $2°C$ increases

With the model, we also predict the range of years in which the relative temperature is $1.25°C$, $1.5°C$ and $2°C$ and we include them in Table 1.

Table 1: Temperature results from the simple model

$°C$	Lower Bound	Mean	Upper Bound
1.25	2025	2032	2039
1.5	2033	2040	2048
2	2046	2055	2065

According to a paper published on global change biology in 2013 [6] and a report by the IPCC [7], the globally accepted maximum temperature of $2°C$ above pre-industrial

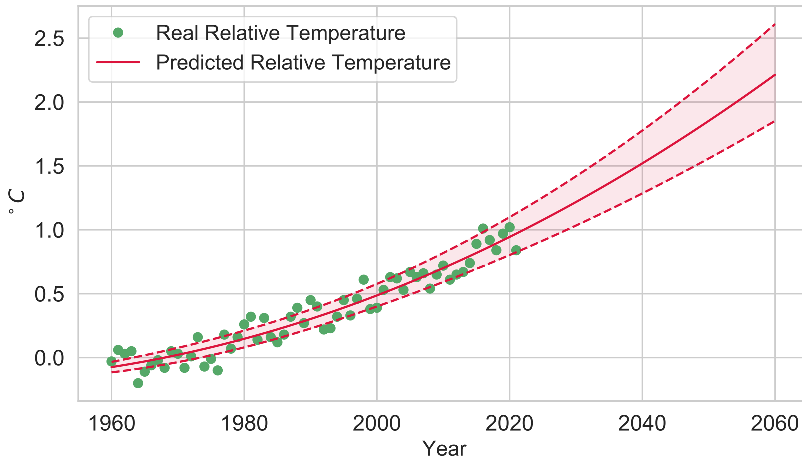

Figure 7: Predicted relative temperatures

revolution temperatures is not undesirable. Ideally, temperatures should stay under a $1.5°C$ difference. From the results, we can see that the temperature will rise to the optimal $1.5°C$ limit around the 2030's and 40's. This suggests that if the concentration grows as predicted, the consequences of global warming will start to compound.

3.3.2 Limitation of the temperature predictions

In Figure 7 we see that as time increases, the upper and lower bounds diverge. Although the data is quite reliable, the difference between the upper and lower bounds become so large that the model stops being effective at giving precise temperature predictions. Since earth is so sensitive to temperature changes we can say that the predicted temperature values become a weak predictor of temperature when the different between upper and lower bounds of temperature becomes greater than $0.5°C$. In this model we can see that this occurs around the 2040's and 50's.

4 Model 2 - A Statistical Model

4.1 What is the statistical model?

In the simple model we only fit a model to the historical CO_2 concentration without considering the root causes. In this model, we consider the causal relations illustrated in Figure 8. We believe that the global economic development and emission reduction effort are the competing forces that largely determine the CO_2 emissions, which then lead to the changes of atmosphere CO_2 concentrations that further causes temperature changes. We use the annual global GDP as the indicator of global economic development, and use carbon intensity, defined as the emission per dollar of GDP, to indicate the result of the emission reduction effort.

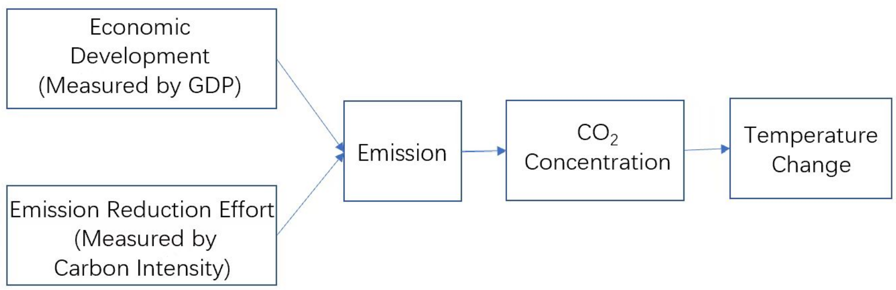

Figure 8: The causal relations of the statistical model

4.2 Modeling emission

Let E_t and G_t denote the global emission and global GDP of year t. The data are available in the period of 1990 to 2020 from the World Bank's website. Let I_t denote the carbon intensity of year t and it is defined as $I_t = E_t/G_t$. We plot the annual GDPs and carbon intensities in Figure 9. Based on the plots and the meaning behind the G_t and E_t, we make the following assumption:

Assumption 2. *GDP and carbon intensity grow exponentially in time.*

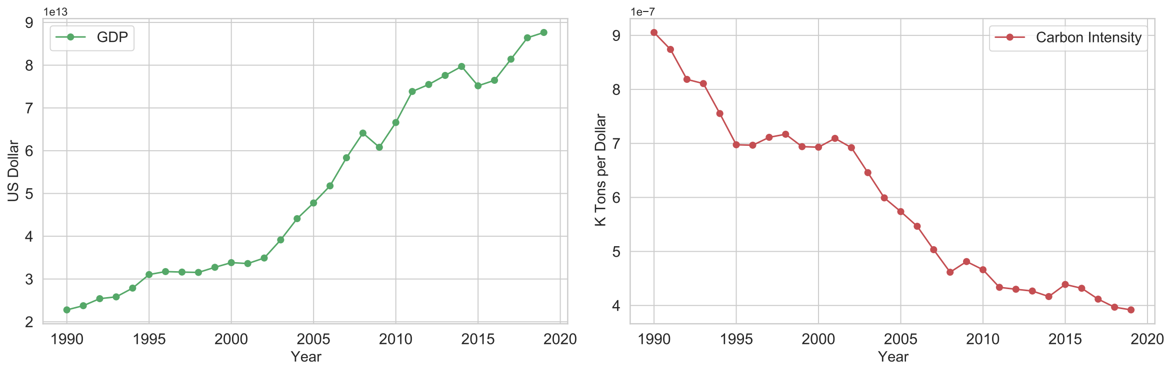

Figure 9: GDP and carbon intensity from 1990 to 2020

Then, we propose the following models of annual GDP and carbon intensity:

$$\begin{aligned} \ln G_t &= \alpha_0 + \alpha_1 t, \\ \ln I_t &= \beta_0 + \beta_1 t. \end{aligned}$$

Then, we have the following model of the annual emission

$$E_t = G_t \cdot I_t = e^{(\alpha_0+\beta_0)+(\alpha_1+\beta_1)t}. \tag{3}$$

We first conduct linear regressions of $\ln G_t$ and $\ln I_t$ with respect to t. The fitted lines are

$$\begin{aligned} \ln G_t &= -69.8417 + 0.0505\,t, \\ \ln I_t &= 45.2556 + -0.0297t. \end{aligned}$$

The R^2 values are 0.968 and 0.955, and the p-values of the coefficients are all very close to 0, indicating the the linear models fit the data very well. We plot the results of the linear regression in Figure 10, where the left panel is the result of GDP and the right panel is the result of carbon intensity.

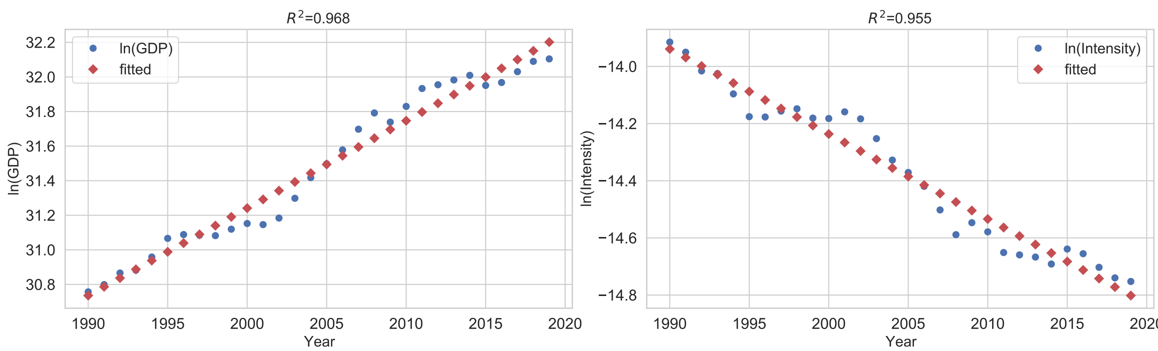

Figure 10: Linear models of Ln(GDP) and Ln(Intensity) to time

Then, we may predict the future annual emissions using Equation (3). We plot the predicted mean values and the lower and upper bounds of the 95% confidence intervals in Figure 11.

4.3 From emission to concentration

Notice that part of the CO_2 emission stay in the atmosphere, thus increasing the atmosphere CO_2 concentration. Therefore, it is reasonable to study the relation between E_t and ΔC_t. We plot ΔC_t with respect to E_t in Figure 12. Again, following the theory of Ockham's razor, we decide that a linear model is valuable as it effectively models the data with the fewest parameters. Then, we make the following assumption

Assumption 3. *ΔC_t is linear in E_t.*

We conduct a linear regression analysis of ΔC_t and E_t, the fitted line is shown on Figure 12. The resulted equation from the linear regression is

$$\Delta C_t = -0.1320 + 7.679 \times 10^{-8}\,E_t. \tag{4}$$

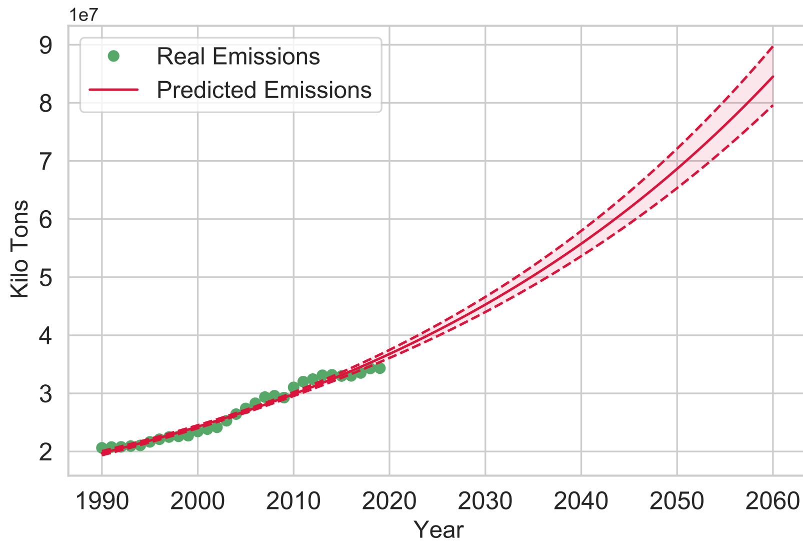

Figure 11: Predicted emissions

Even though the R^2 valuPe is only 0.380, the p-values of the coefficients are very close to 0, suggesting that this linear model is still statistically significant despite the low R^2 value, which may be caused by the randomness in the data.

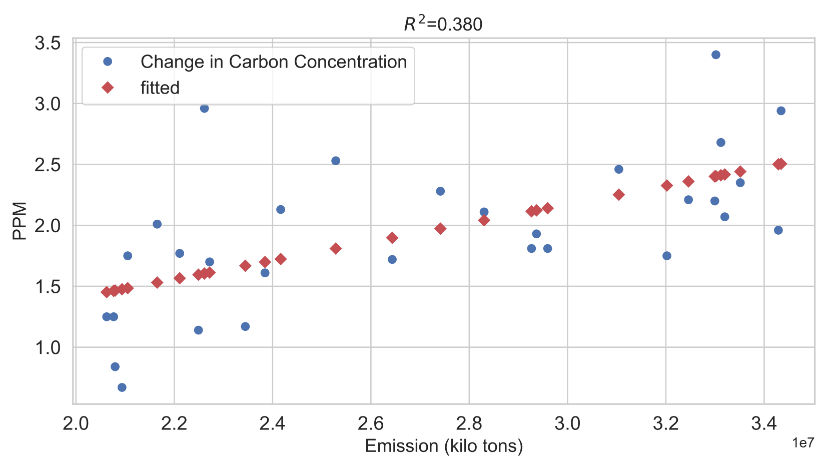

Figure 12: Linear model of ΔC_t to E_t

From this we can recursively predict future C_t values from predicted E_t using the equation *as* as seen in Figure 13. From Figure 13 we can see that, after taking into consideration the GDP and carbon intensity, we expect the CO_2 concentration to be between 489.65 ppm and 573.30 ppm with the mean of 529.24 ppm at 2050, while we expect the concentration to be between 752.60 ppm and 1274.45 ppm with the mean of 990.99ppm at 2100. The concentration of 685 ppm predicted by the OECD is still significantly beyond the 95% confidence interval given by our data. In fact, examining

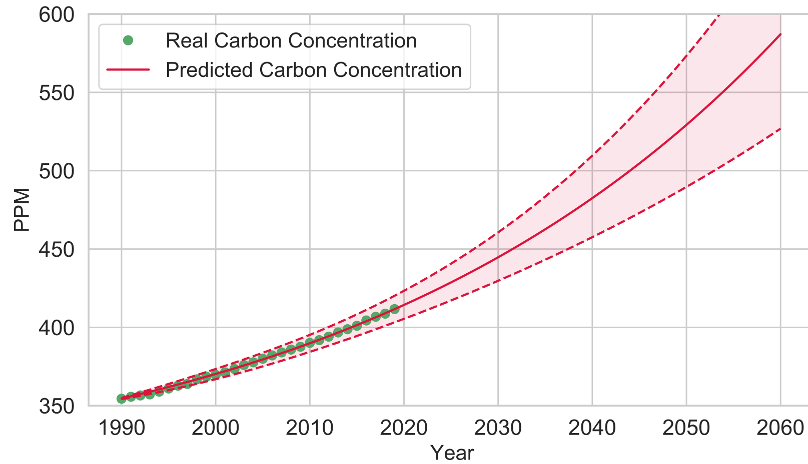

Figure 13: Predicted CO_2 concentration

our graph, we expect the CO_2 concentration to be 685 ppm at the earliest year of 2064 and latest year of 2091. Therefore, from our statistical model, we do not agree with the OECD's prediction of 685 ppm at 2050.

4.4 Temperature predictions from the statistical model

With the predicted CO_2 concentrations from the statistical model, we can use the model between the CO_2 concentration and relative temperature (Equation 1) established in Section 2 to predict the relative temperatures in the future. We take the mean values and the upper and lower bounds of the predicted concentrations to calculate the mean values and the upper and lower bounds of the predicted relative temperatures, respectively. This is illustrated in Figure 14.

With the model, we also predict the range of years in which the relative temperature is $1.25°C$, $1.5°C$ and $2°C$ and we include them in Table 2. The increase in temperature predicted by the statistical model (i.e., Model 2) is slightly higher than that of the simple model (i.e., Model 1). From the results, we can see that the temperature will rise to the optimal $1.5°C$ limit around 2030's and 40's. This again suggests that we must do something to slow down the speed of the temperature increase.

Table 2: Temperature results from the statistical model

$°C$	Lower Bound	Mean	Upper Bound
1.25	2024	2030	2038
1.5	2030	2037	2046
2	2039	2048	2061

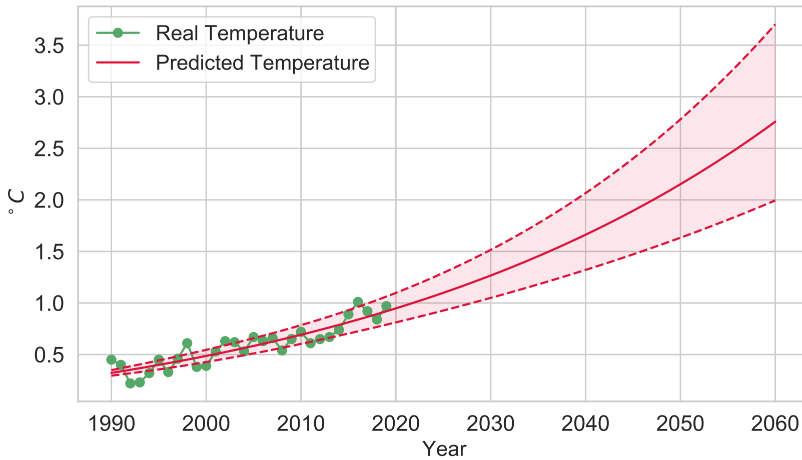

Figure 14: Predicted temperature

4.5 Limitation of the statistical model

The statistical model takes into consideration the causal relations illustrated in 8. It is more meaningful than the simple model. However, it still has two limitations. First, the causal relation between the emission and the change in atmosphere CO_2 concentration is purely statistical and no physics laws are considered to support the linear-regression model. Second, while the statistical model is clear and powerful in predictions, it does not provide a single formula that puts all factors together so that the impacts of different factors are more transparent.

5 Model 3: A Mechanistic Model

5.1 What is the mechanistic model?

In the statistical model, the causal relation between the emission and the change in atmosphere CO_2 concentration is purely statistical and no physics laws are considered to support the linear-regression model. In the mechanical model, we take into consideration the possible sinks of the environment as how they are influenced by the change in concentration of CO_2 in the atmosphere. This is because we know that certain sinks, such as land and oceans, change the amount of CO_2 they absorb based on the amount of CO_2 in the atmosphere. As such, we can take this idea into consideration and implement it into our statistical model to form a mechanistic model.

To formulate the mechanistic model, we convert the statistical model into a more abstract differential-equation form. Let

$$\begin{aligned}\frac{dG}{G_t} &= \mu_G dt,\\ \frac{dI_t}{I_t} &= -\mu_I dt,\\ \frac{dE_t}{E_t} &= \frac{dG}{G_t} + \frac{dI_t}{I_t} = (\mu_G - \mu_I)dt,\end{aligned}$$

where μ_G denotes the annual growth rate of GDP and μ_I denotes the annual reduction rate of the carbon intensity. From there we can directly calculate the amount of CO_2 emitted,

$$E_t = E_0 e^{(\mu_G-\mu_I)t}. \tag{5}$$

Then, we model

$$dC_t = \alpha_0 \left(\frac{C_t}{C_0}\right)^{1-\gamma} dE_t \tag{6}$$

with $0 < \gamma \leq 1$, which implies that:

- Compared to the pre-industrial revolution time, i.e., $t = 0$, the percentage of a unit of emission staying in the atmosphere increases with C_t, because the absorbing abilities of the ocean and the land decrease as their CO_2 level increases and, therefore, more CO_2 is left in the atmosphere.
- Also, $\frac{dC_t}{C_t}$ reduces in C_t, meaning that the percentage increases of a unit of emission reduces as the concentration level increases.
- When $\gamma < 1$, the CO_2 concentration increases faster than the emission, which is the scenario that scientists warn about.
- When $\gamma = 1$, it becomes a special case of the linear model that we use in the statistical model.

Given Equations (5) and (6), we have

$$dC_t = \alpha_0 \left(\frac{C_t}{C_0}\right)^{1-\gamma} (\mu_G - \mu_I)E_0 e^{(\mu_G-\mu_I)t} dt.$$

After solving the differential equation with the boundary condition $C_t = C_0$ when $t = 0$, we find

$$C_t = \left\{C_0^\gamma + \gamma C_0^{\gamma-1}\alpha_0 E_0 \left[e^{(\mu_G-\mu_I)t} - 1\right]\right\}^{\frac{1}{\gamma}}.$$

This model gives a clear picture on how economic development (indicated by μ_G), carbon reducing effort (indicated by μ_I) and the absorbing mechanism (indicated by γ)

affect the overall CO_2 concentration. In particular, the economic development pushes up the concentration, the carbon reducing effort pushes down the concentration, while the absorbing mechanism has a nonlinear impact in the sense that a smaller γ will cause the concentration to grow faster.

5.2 Modeling change in CO_2 concentration

While the mechanistic model has a clear formula of C_t, it is difficult to fit the data and to predict the future CO_2 concentration because it is insensitive to the choice of γ. Motivated by Equation (6), we propose the following model to fit the data:

$$\Delta C_t = \gamma_0 + \gamma_1 \left(C_{t-1}/C_0\right)^{1-\gamma} E_t, \tag{7}$$

utilizing the constants from the previous statistical model of Equation (4), -0.1320 as γ_0 and 7.679×10^{-8} as γ_1. We vary the value of γ to minimize the squared errors and find the optimal γ value is 0.969. From the fitted model, we can recursively predict the future C_t values from the predicted E_t and plot them in Figure 15.

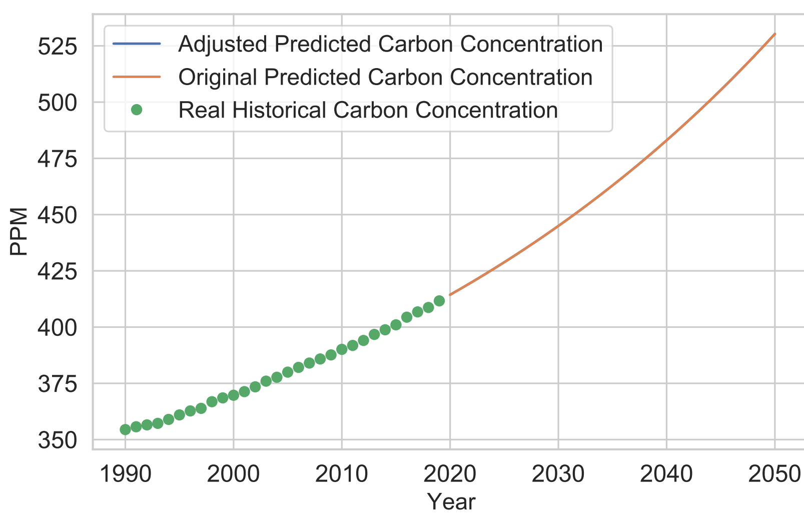

Figure 15: Predict carbon concentration

From Figure 15, we can see that it predicates the CO_2 concentration to be 530.31 ppm at 2050. It can also be computed that the CO_2 concentration at 2100 is 1002.85 ppm. Like before, this model still does not match the OECD's prediction of 685 ppm by 2050. In fact, our model predicts that it will only reach 685 ppm by 2074. Therefore, it can be seen that even our mechanistic model still does not agree with the OECD's predictions.

When we fit Equation (7), we notice that the resulted error is insensitive to the value of γ. However, the prediction value is quite sensitive to γ. This is because the effect of γ is nonlinear and it will only be significant when the CO_2 concentration level is high. Therefore, we suspect that the optimal γ value of 0.969 may have a lot of uncertainty in it. Therefore, we vary the value of γ, and observe and compare other possible outcomes of the CO_2 concentrations. This is shown in Figure 16.

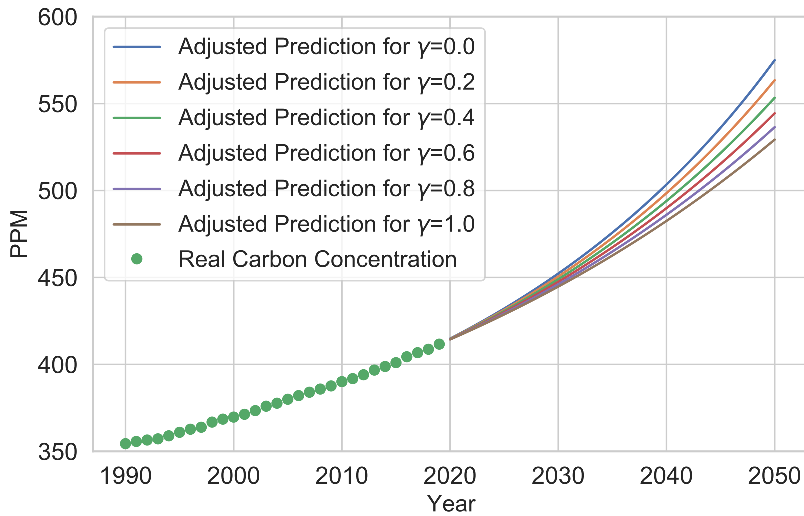

Figure 16: Predicted carbon concentration with varying γ values

Even when the predicted CO_2 concentration is as highest as it can be with γ being equal to 0, we can see that the CO_2 concentration predicted is only 574.93 ppm, still a ways off from 685 ppm. This means that even taking into account the possibility that our γ prediction is inaccurate, from the range shown, our models still can't reach the prediction from the OECD.

5.3 Temperature prediction from the mechanistic model

Using the predicted CO_2 concentration from the mechanistic model, we can use the same CO_2 concentration to relative temperature model (Equation 1) from Section 2 to predict the possible relative temperature in the future. This is shown in Figure 17.

From this we can see that the predicted year at which our model says that it will reach a relative temperature of 1.25, 1.5, and 2 are in 2030, 2037 and 2047, respectively. Comparing these years to the mean years of Table 2 from our statistical model, we can see that there is no significant change. This means our mechanistic model still supports the idea that the temperature will rise to the optimal $1.5°C$ within the 2030's to 40's range.

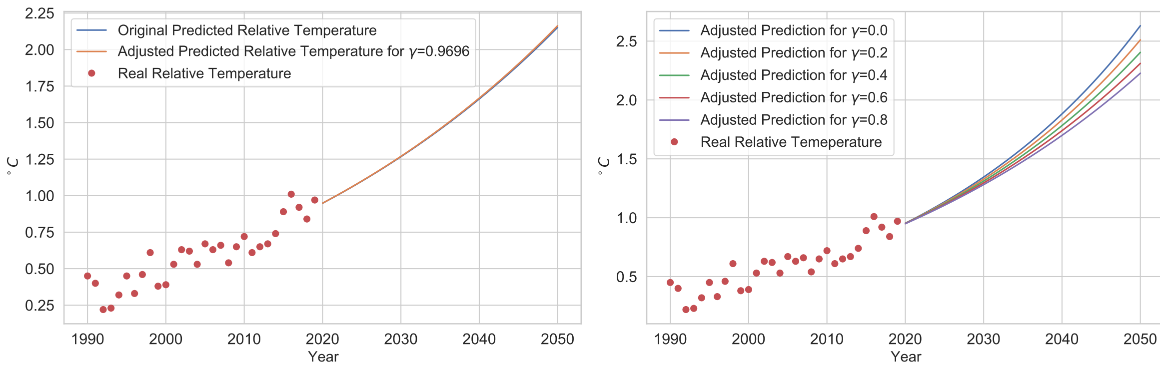

Figure 17: Predicted temperature with optimal γ values

6 A Better Scenario: Carbon Control Policies

Based on the three models that we created, it seems very unlikely that we will be able to limit the temperature increase to $1.5°C$ by 2050 and $2°C$ by 2100. However, all three models assume that we follow the current trends and this may not be true. In recent years, recognizing the need of immediate reactions to global warming, many countries have adopted policies in which they aim to reduce their CO_2 emissions and reach net zero emissions (carbon neutrality) by a certain year. In this section, we aim to predict the future CO_2 concentration and future temperature in the scenario that every country is successful in implementing their carbon control policies and reach their carbon reduction targets (which we call a better scenario).

6.1 Modeling emission in the better scenario

According to the Intergovernmental Panel on Climate Change (IPCC) of the United Nations, the top three emitting regions are China, USA and European Union (EU), each accounting for 30%, 15% and 9% of the global emission, respectively. Therefore, in this section, we first analyze the carbon control policies of these three regions. Table 3 lists promised times for the peak of carbon emissions and the carbon neutrality by the governments of the three regions. USA and EU have already reached the peak of carbon emissions, and they announced respective emission targets by 2030 and promised to achieve carbon neutrality by 2050. China promised to reach the peak by 2030 and achieve carbon neutrality by 2060. To help understand these policies, we also create a figure to illustrate (Figure 18).

To model the global emission under these policies, we make the following assumptions to simplify the analysis.

Assumption 4. *CO_2 values are in net emissions.*

Table 3: Top 3 emitters' carbon control goals

Country	Goal
China	Peak emissions (11.7 million kt) by 2030 and carbon neutrality by 2060
US	5.7 million kt by 2030 and carbon neutrality by 2050
EU	3.4 million kt by 2030 and carbon neutrality by 2050

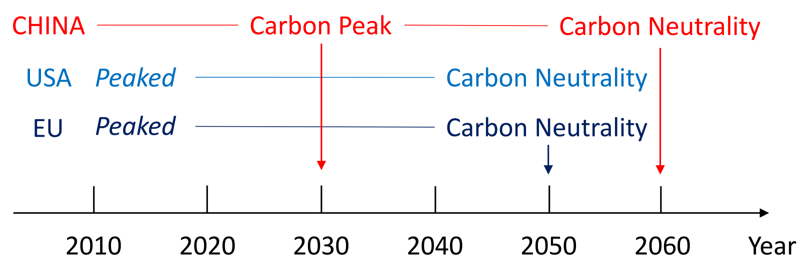

Figure 18: Carbon control timeline

We do not have data for the amount of CO_2 absorbed by sinks (such as land and oceans). Therefore, we assume that the CO_2 values given are for net CO_2 (including human emissions and natural sinks) to predict the CO_2 emissions for when countries reach "net zero" emissions.

Assumption 5. *The emissions values change linearly to their targets.*

Since the current emissions data do not reflect their respective countries' emissions reduction plan and, by Ockham's razor, we believe that just linearly increasing or decreasing the emissions data to match the target emissions is sufficient in showing the overall global emissions trend.

Assumption 6. *The other countries' CO_2 emission patterns will follow the same pattern as the top 3 emitters.*

The top three emitters of CO_2 right now constitute for 54% of the entire world's emissions. Additionally, the combination of China, Eu, and the US offer a thorough representation of the general carbon trends of the world, with China being a developing country who has yet to peak in emissions, while the Eu and US are already developed and planning to reduce emissions post industrialization. We assume that the combined values of these entity offer a thorough representation of carbon emission goals for all the countries in the world. Therefore, using their data offers us an accurate insight into the future of emission planning.

Based on these assumptions, we may predict the future CO_2 emissions of China, USA and EU based on their respective targets and predict the global CO_2 emissions using the following equation:

$$E_{Global,t} = \frac{(E_{China,t} + E_{USA,t} + E_{EU,t}) \times 100}{54},$$

where $E_{China,t}$, $E_{USA,t}$, and $E_{EU,t}$ denote the CO_2 emissions of China, USA and EU in year t, and $E_{Global,t}$ denotes the global CO_2 emissions in year t. The predicted global CO_2 emissions under the better scenario are plotted in Figure 19.

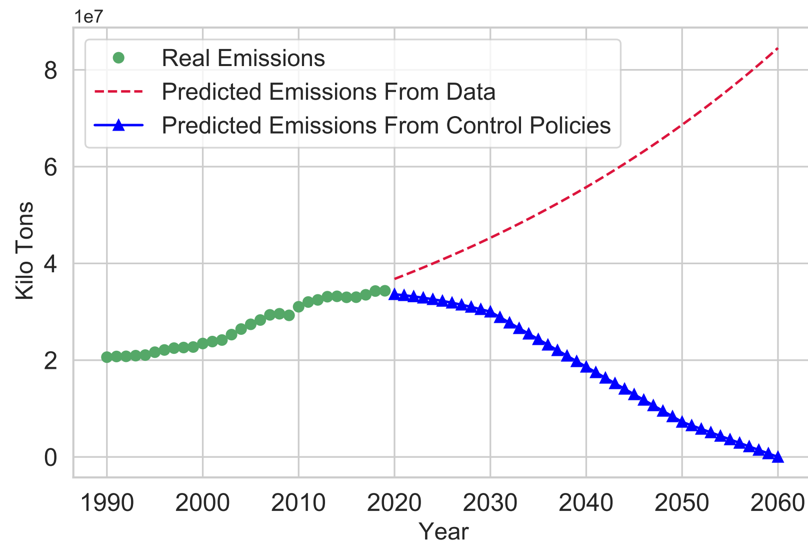

Figure 19: Predicted global emissions in the better scenario

From the figure we see that the predicted global CO_2 emissions take a completely different turn from what we predicted under the original scenario (i.e., based on the historical data only) and they reach 0 by 2060.

6.2 Concentrations and temperature in the better scenario

By using the global emissions data in Figure 19 and plugging it into model 2, we can see the difference in carbon concentration in Figure 20 where the carbon concentration stops increasing and the lower bound even starts decreasing.

By then using the concentration data, we can use Equation (1) to find the temperature as seen in Figure 21.

After seeing Figures 20 and 21 we can see that under this better scenario where we achieve the world's carbon control targets the atmospheric CO_2 concentration never exceeds 500 ppm and subsequently the temperature never exceeds a $2°C$ difference compared to the pre-industrial revolution temperature. This means that under this scenario we will not have to experience the compounding effects of global warming.

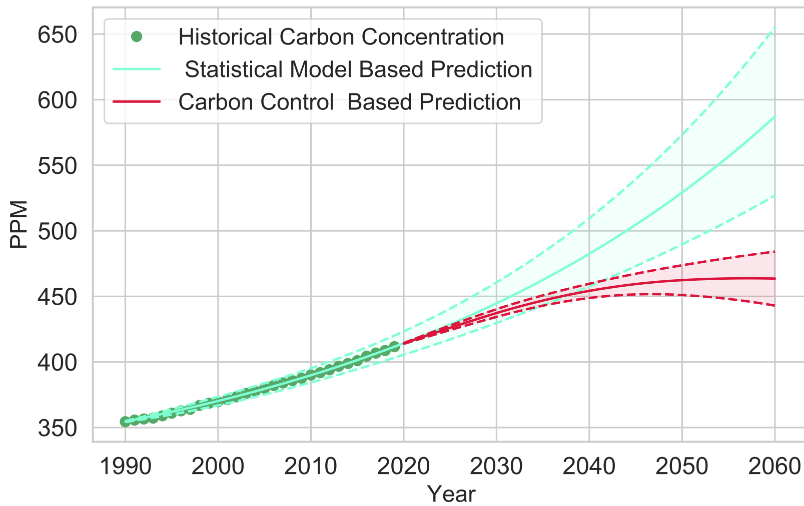

Figure 20: Predicted carbon concentration

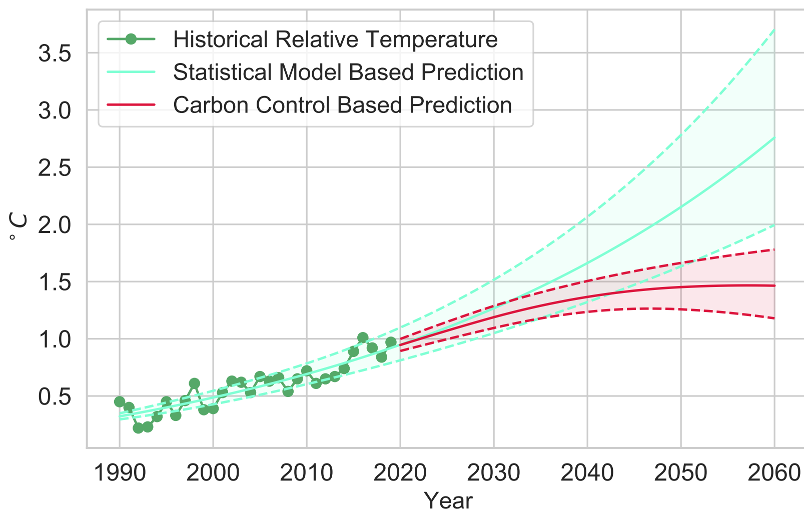

Figure 21: Predicted temperature

6.3 Limitations of the better-scenario model

Although the model of what would happen under a better scenario gives us a general idea of how global carbon concentration and temperatures could change for the better in the near future, it raises the question whether this model can represent the future emissions well enough. Due to the limited data and knowledge on carbon emissions plans we had to only use the data from the three biggest emitters to predict the future emissions. The problem with this is that the three biggest emitters: China, the US, and the EU, are developed countries. Currently there are many developing regions such as the continents of Africa and South-America. Developing countries typically have greatly increasing carbon emissions annually. For example, from the World Bank's data, we can see that in past 30 years, China's CO_2 emissions has increased 5 fold. If similar emissions patterns occur in other developing countries, this could be a serious problem.

7 Conclusion

We created three models to understand and predict changing rates of CO_2 concentration and its relationship with temperature. Our simple model (#1) only takes into account previous CO_2 concentration to predict the future concentration. It has small confidence intervals suggesting that it is reliable in predicting concentration and temperature for a long period of time. However, the simple model fails to account external factors that may impact what we're measuring. This is why we created the statistical model (#2). This model uses GDP and intensity to predict carbon concentration and temperature. This model is a better predictor of concentration and temperature in the short run. However, due to the large number of variables, the confidence intervals grow to be very large very quickly resulting in the model being unreliable for predictions in the long run. Last, we believe that the diminishing effectiveness of carbon sinks plays a role in carbon concentration and temperature. This is why we created the mechanistic model (#3) which let us find a large variety of predicted carbon concentrations while carbon control policies are accounted.

Just knowing the models is not enough. We now need to understand the implications and what we could do about them. We have seen from model 1 that the carbon concentration in the atmosphere is increasing at a compounding rate. This growth in carbon concentration is causing a similar growth pattern to occur in the temperature models. We know that temperature reaches a critical mass around $1.5°C$ to $2°C$ above pre-industrial revolution temperatures. Although each of the three models uses a different method of calculation and have confidence intervals of varying ranges, they still all show that the global temperature will reach critical mass in the next 20 to 30 years. This makes it evident that something has to change. Our better scenario model (#3) finds that under the circumstance that the entire world achieves their carbon reduction goals, we can be successful in keeping the global temperature increase under $2°C$.

Now that we know things need to change. Remember a journey of a thousand miles begins with a single step. Be active in trying to reduce your carbon footprint. According to The Nature Conservancy, roughly 36% of the average American's carbon emissions comes from travelling. Therefore, using public transit or carpooling with others is a great way to reduce your carbon footprint. Furthermore, reduce your airplane flight as it significantly increases your carbon footprint [8]. Lastly, it is paramount that you make sure that your household electricity comes from a safe and renewable source.

8 References

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A Appendix: Python Code of Regression and Visualization

Below is an example of the python code that that we use to do regression and visualize the results.

Regression and Visualization For the Simple Model

```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
import pandas as pd
import statsmodels.api as sm
import seaborn as sns
import statsmodels
sns.set()

data = pd.read_csv("Data.csv")

x = data[['Year']][2:]
y = data['Change in Carbon Concentration'][2:]
x = sm.add_constant(x)
model = sm.OLS(y, x).fit()

future_x = np.arange(1960, end+1)
future_x = sm.add_constant(future_x)
pred_changes = model.predict(future_x)
results = model.get_prediction(future_x)
df_results = results.summary_frame()
pred_changes = df_results["mean"]
pred_changes_low = df_results["mean_ci_lower"]
pred_changes_high = df_results["mean_ci_upper"]
future_concentrations = sm.add_constant(315.98 + np.cumsum(pred_changes))
future_concentrations_low = sm.add_constant(315.98 + np.cumsum(pred_changes_low))
future_concentrations_high = sm.add_constant(315.98 + np.cumsum(pred_changes_high))

x = data[['Carbon Concentration']][1:]
y = data['Relative Temperature'][1:]
x = sm.add_constant(x)
model = sm.OLS(y, x).fit()

results = model.get_prediction(future_concentrations)
df_results = results.summary_frame()
temps = df_results["mean"]
results = model.get_prediction(future_concentrations_low)
df_results = results.summary_frame()
temps_low = df_results["mean_ci_lower"]
results = model.get_prediction(future_concentrations_high)
df_results = results.summary_frame()
temps_high = df_results["mean_ci_upper"]

fig, ax = plt.subplots(figsize = (9, 5))
plt.plot(data["Year"][2:], data["Relative Temperature"][2:], "go",
    label = "Real Relative Temperature")
plt.plot(future_years, temps, "-", color='crimson', label="Predicted Temperature")
plt.plot(future_years, temps_low, "--",color='crimson')
plt.plot(future_years, temps_high, "--",color='crimson')
plt.fill_between(future_years, temps_low, temps_high, alpha=.1, color='crimson')
```